Logical Foundations of Agents (4)

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KARO logic

- Knowledge & Belief:
  - – epistemic logic
- Abilities, Results & Opportunities:
  - – dynamic logic
- Modalities for Wishes & Goals

Epistemic states

- in epistemic logic:
  - R equivalence relation
  - {w' | R(w,w')} equivalence class of w
  - determines *epistemic state*

Epistemic logic

- $K\varphi \rightarrow \varphi$
- $K\varphi \rightarrow KK\varphi$ *knowledge*
- $\neg K\varphi \rightarrow K\neg K\varphi$

- $\neg B\bot$
- $B\varphi \rightarrow BB\varphi$ *belief*
- $\neg B\varphi \rightarrow B\neg B\varphi$

The logical omniscience problem

(LO1) øφ ∧ ø(φ ‘ ψ) ‘ øψ

(Closure under implication).

(LO2) • φ ’ • øφ

(Belief of valid formulas).

(LO3) • φ ‘ ψ ’ • øφ ‘ øψ

(Closure under valid implication).

Logical omniscience (2)

(LO4) **• φ – ψ ' • øφ – øψ**

(Belief of equivalent formulas).

(LO5) **(øφ ∧ øψ) ‘ ø (φ ∧ ψ)**

(Closure under conjunction).

(LO6) **øφ ‘ ø (φ Ú ψ)**

(Weakening of belief).

(LO7) **øφ ‘ ¬ø¬φ**

(Consistency of beliefs).

Dynamic Logic

- Syntax
  - Operator [α] with reading:
  - $[\alpha]\varphi$: after execution of α it holds that φ
  - $<\alpha>\varphi = \neg[\alpha]\neg\varphi$
- Semantics
  - Accessibility relation R_α for every action α
    - $R_{\alpha;\beta}$ = R_α ã R_β
    - $R_{\alpha+\beta} = R_\alpha \cup R_\beta$
    - $R_{\alpha*} = R_\alpha{}^*$

Dynamic Logic

- Interpretation formulas
  - M,s • [α]φ ⇔ for all s' with R_{α}(s,s'): M,s' • φ
  - M,s • <α>φ ⇔ for some s' with R_{α}(s,s'): M,s' • φ

Dynamic Logic

- Basic property (K)
  - $[\alpha](\varphi \rightarrow \psi) \rightarrow ([\alpha]\varphi \rightarrow [\alpha]\psi)$

  Structure of actions
  - $[\alpha_1 ; \alpha_2]\varphi \leftrightarrow [\alpha_1]([\alpha_2]\varphi)$
  - $[\alpha_1 + \alpha_2]\varphi \leftrightarrow [\alpha_1]\varphi \wedge [\alpha_2]\varphi$
  - $[\alpha^*]\varphi \rightarrow \varphi$
  - $[\alpha^*]\varphi \rightarrow [\alpha][\alpha^*]\varphi$
  - $[\alpha^*](\varphi \rightarrow [\alpha]\varphi) \rightarrow (\varphi \rightarrow [\alpha^*]\varphi)$

KARO

- **K: Knowledge (& Belief)**
  - epistemic logic:
    - Knowledge K_i : the logic S5
    - Belief B_i : the logic weak S5
- **A: Abilities**
  - ability operator A_i

KARO

- **R: Results**
  - – dynamic logic ("multi-modal K"):
    - **$[do_i(\alpha)]\varphi$**

- **O: Opportunities**
  - – dynamic logic:
    - **$\langle do_i(\alpha)\rangle true$**

KARO

Interaction:

$K_i[do_i(\alpha)]\varphi \rightarrow [do_i(\alpha)]\, K_i\, \varphi$ perfect recall

$[do_i(\alpha)]\, K_i\, \varphi \rightarrow K_i[do_i(\alpha)]\varphi$ no learning

in principle expressible.
Assuming them makes system intractable

KARO: correctness, feasibility

- $\text{Correct}_i(\alpha, \varphi) = \langle do_i(\alpha)\rangle\varphi$
- $\text{Feasible}_i(\alpha) = A_i\alpha$

- $\text{PracPoss}_i(\alpha,\varphi) = \text{Correct}_i(\alpha,\varphi) \ldots \text{Feasible}_i(\alpha)$

- $\text{Can}_i(\alpha,\varphi) = K_i\text{PracPoss}_i(\alpha,\varphi) = K_i(\langle do_i(\alpha)\rangle\varphi \ldots A_i\alpha)$
- $\text{Cannot}_i(\alpha, \varphi) = K_i\neg\text{PracPoss}_i(\alpha, \varphi)$

KARO : Can, implementability

- some properties:
  - $Can_i(\alpha_1; \alpha_2, \varphi) \leftrightarrow Can_i(\alpha_1, PracPoss_i(\alpha_2, \varphi))$
  - $Can_i(\alpha_1; \alpha_2, \varphi) \rightarrow \langle do_i(\alpha)\rangle Can_i(\alpha_2, \varphi)$,

  if α_1 *accordant*, i.e. $K_i\, [do_i(\alpha)]\varphi \rightarrow [do_i(\alpha)]K_i\varphi$

- $Impl_i\varphi$ = "$PracPoss_i(a_1; ...; a_k, \varphi)$) for some atomic actions $a_1, ..., a_k$"

KARO : informational attitudes

- Belief types & belief revision in an agent-oriented setting:
  - $K = B_i^k$ (certain) knowledge
  - B_i^o belief by observation
  - B_i^c belief by communication
  - B_i^d belief by default

KARO: informational attitudes

- $Agn_i^x = \neg B_i^x \varphi \ldots \neg B_i^x \neg \varphi$ (x £ {k, o, c, d})
- $Saw_i \varphi = Agn_i^k \varphi \ldots B_i^o \varphi$
- $Heard_i \varphi = Agn_i^o \varphi \ldots B_i^c \varphi$
- $Jumped_i \varphi = Agn_i^c \varphi \ldots B_i^d \varphi$

KARO : informational attitudes

- validities:
  - $<do_i(observe\ \varphi)>\neg Agn_i^{o}\varphi$
  - $\varphi \rightarrow <do_i(observe\ \varphi)> B_i^{o}\varphi$
  - $\neg\varphi \rightarrow <do_i(observe\ \varphi)> B_i^{o}\neg\varphi$
  - $\varphi \ldots Agn_i^{k}\varphi \rightarrow <do_i(observe\ \varphi)>Saw_i\varphi$
  - $\neg\varphi \ldots Agn_i^{k}\varphi \rightarrow <do_i(observe\ \varphi)>Saw_i\neg\varphi$
  - $\varphi \ldots (Heard_i\neg\varphi$ Ú $Jumped_i\neg\varphi) \rightarrow$

    $<do_i(observe\ \varphi)>Saw_i\varphi$

KARO : Agents that change their mind

- α: contract_ φ, expand_ φ, revise_ φ
  - $\langle do_i(\alpha)\rangle T$
  - $\langle do_i(\alpha)\rangle \varphi \rightarrow [do_i(\alpha)] \varphi$
  - $[do_i(\alpha;\alpha)] \varphi \leftrightarrow [do_i(\alpha)] \varphi$

KARO : Agents that change their mind

- α: contract_ φ,
  - $[do_i(\alpha)]B_i\psi \rightarrow B_i\psi$
  - $\neg B_i\varphi \rightarrow ([do_i(\alpha)]\, B_i\psi \leftrightarrow B_i\psi)$
  - $\neg K_i\varphi \rightarrow [do_i(\alpha)]\, \neg\, B_i\psi$
  - $B_i\varphi \rightarrow ([do_i(\text{contract\_}\varphi\,;\ \text{expand\_}\varphi)]\, \psi \leftrightarrow \psi)$

KARO : Agents that change their mind

- Levi: revise_ φ = contract_ φ; expand_ φ
- Baltag, Gerbrandy, Lomuscio: our approach is
  - ONE AGENT!
- Problems when φ is not propositional
  - p ... ¬Bp is consistent, but how to revise your belief with it?

KARO : informational attitudes

- $B_j^{d}\varphi \ldots \neg D_{i,j}\varphi \rightarrow (\langle do_j(\text{inform }(\varphi, i)\rangle \chi \leftrightarrow \chi)$
- $D_{i,j}\varphi \ldots B_j^{o}\varphi \rightarrow \langle do_j(\text{inform }(\varphi, i)\rangle B_i^{c}\varphi$
- $D_{i,j}\varphi \ldots B_j^{o}\varphi \ldots Agn_i^{o}\varphi \rightarrow$

  $\langle do_j(\text{inform }(\varphi, i))\rangle Heard_i\varphi$
- $D_{i,j}\varphi \ldots Heard_j\varphi \ldots Agn_i^{d}\varphi \rightarrow$

  $\langle do_j(\text{inform }(\varphi, i))\rangle Heard_i\varphi$
- $D_{i,j}\varphi \ldots Heard_j\varphi \ldots \neg Agn_i^{d}\varphi \rightarrow$

  $(\langle do_j(\text{inform }(\varphi, i))\rangle \chi \leftrightarrow \chi)$

[$D_{i,j}\varphi$: dependency of agent i on agent j w.r.t. φ]

KARO : informational attitudes

- Further validities:
  - Default(φ) ... $Agn_i^d \varphi \rightarrow$

    $\langle do_i(try\_jump\ \varphi)\rangle Jumped_i \varphi$
  - Default(φ) ... $\neg Agn_i^d \varphi \rightarrow$

    $(\langle do_i(try\_jump\ \varphi)\rangle \chi \leftrightarrow \chi)$

Extended KARO : motiv. att's

- Des_i : desirability operator
- $Goal_i(\varphi) = Des_i\varphi \ldots \neg\varphi \ldots Impl_i\varphi$
- NOT: $\varphi \rightarrow \psi \Rightarrow Goal_i(\varphi) \rightarrow Goal_i(\psi)$
- $CanG_i(\alpha, \varphi) = Can_i(\alpha, \varphi) \ldots Goal_i(\varphi)$

Extended KARO : motiv. att's

- commit_to_α
- $\text{Committed}_i(\alpha)$

- some properties:
  - $\text{CanG}_i(\alpha, \varphi) \rightarrow \langle \text{do}_i(\text{commit\_to\_}\alpha) \rangle \text{Committed}_i(\alpha)$
  - $\text{Committed}_i(\alpha_1 ; \alpha_2) \rightarrow \text{Committed}_i(\alpha_1) \ldots$ $K_i[\text{do}_i(\alpha)]\text{Committed}_i(\alpha_2)$

Extended KARO : motiv. att's

- some properties:
  - $\text{Committed}_i(\alpha) \rightarrow K_i\ \text{Committed}_i(\alpha)$
  - $(\text{Committed}_i(\text{ifb then } \alpha_1 \text{else } \alpha_2)$ … $K_i b) \rightarrow \text{Committed}_i(\alpha_1)$
  - $\text{Committed}_i(\alpha) \leftrightarrow (\langle do_i(\text{commit\_to\_}\alpha)\rangle \neg\text{Committed}_i(\alpha))$

Extended KARO

- $\mathbf{PracPoss}_i(\alpha, \varphi) \triangleq \langle \mathrm{do}_i(\alpha) \rangle \varphi \wedge \mathbf{A}_i \alpha$
- $\mathbf{Can}_i(\alpha, \varphi) \triangleq \mathbf{K}_i \mathbf{PracPoss}_i(\alpha, \varphi)$
- $\mathbf{Cannot}_i(\alpha, \varphi) \triangleq \mathbf{K}_i \neg \mathbf{PracPoss}_i(\alpha, \varphi)$

$$\Diamond_i \varphi \triangleq \exists k \in \mathbb{N} \exists a_1, \ldots, a_k \in \mathrm{At}(\mathbf{PracPoss}_i(a_1; \ldots; a_k, \varphi))$$

$$\mathbf{Goal}_i \varphi \triangleq \mathbf{W}_i \varphi \wedge \neg \varphi \wedge \Diamond_i \varphi \wedge \mathbf{C}_i \varphi$$

Reasoning about action & change

- Frame problem and related problems
- Situation calculus, event calculus, fluent calculus
- Planning
- Relation with non-monotonic reasoning

Frame problem(s)

- Persistence problem
  - Assumption of inertia, minimal change
- Qualification problem
  - What are the preconditions of a successful performance of an action?
- Ramification problem
  - What *does* change as an (indirect) result of an action?

Situation calculus (McCarthy)

- *First-order* framework to reason about actions and change
- Situation: actual or hypothetical state of the world at a particular time
- Result(A, s): the new situation after action A has been performed in situation s
- Holds(p, s): the property p is true in situation s

Situation calculus, axiomatisation
Lin & Reiter (Herbrand situations)

- $S0 \neq Result(A, s)$
- $Result(A, s) = Result(A', s') \rightarrow A = A' \wedge s = s'$
- *Induction rule:*

$$\frac{\Phi(S0) \qquad \Phi(s) \rightarrow \forall A[\Phi(Result(A, s))]}{\forall s[\Phi(s)]}$$

Situation calculus

- Some authors don't like the concept of a Herbrand situation, which amounts to a *situation as a sequence of actions performed in an initial situation*
- They prefer to view situations as states, thus reifying the abstract notion of state, including them in our conceptualisation of the world $\Rightarrow$ put states into *object language*

Situation calculus, state semantics

- *States* are mappings from properties to truth-values (or 'fluents' to their values)
- *States* are characterized by:
  - *Uniqueness* property

$$\forall r \forall r'[\forall p[Holds(p,r) \equiv Holds(p,r')] \rightarrow r=r']$$

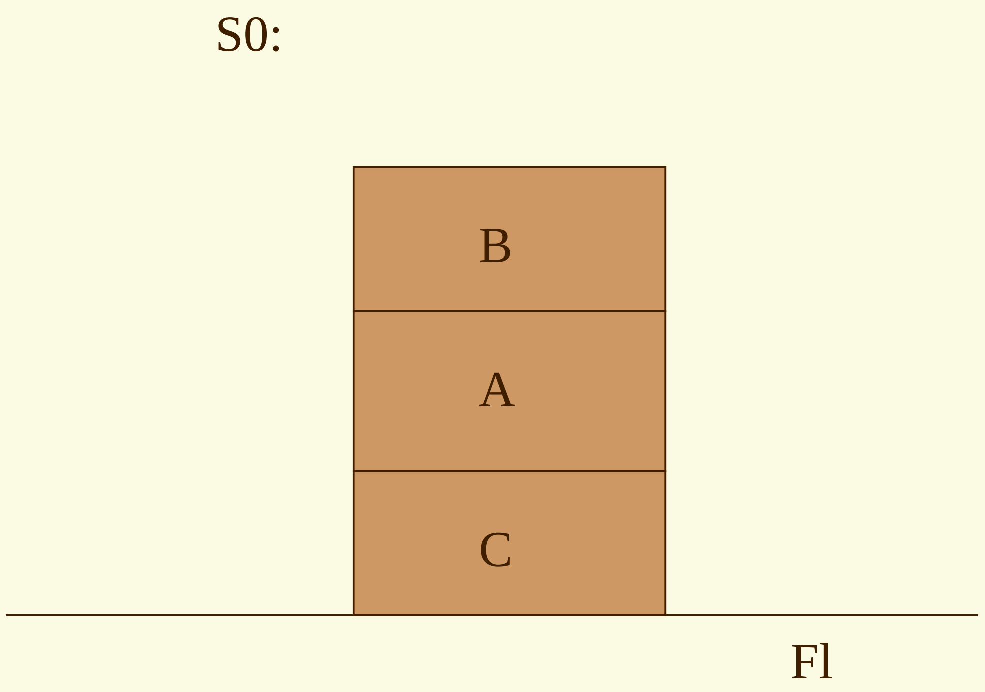
  - *Existence* property

$$\forall r \forall p \exists r'[(Holds(p,r) \equiv \neg Holds(p,r')) \wedge$$

$$\forall p'[p \neq p' \rightarrow (Holds(p',r) \equiv Holds(p',r'))]]$$

Situation calculus, example

- On(B, A, S0)
- On(A, C, S0)
- On(C, Fl, S0)
- Clear(B, S0)
- Clear(Fl, S0)



Sit. Calc., example (ctd)

- Some propositions true of all states:
  - $(\forall x,y,s)[On(x,y,s) \wedge \neg(y = Fl) \rightarrow \neg Clear(y,s)]$
  - $(\forall s)\ Clear(Fl,s)$
- Derived assertions:
  - $\neg Clear(A,S0)$
  - $Clear(Fl,S0)$

Actions in the situation calculus

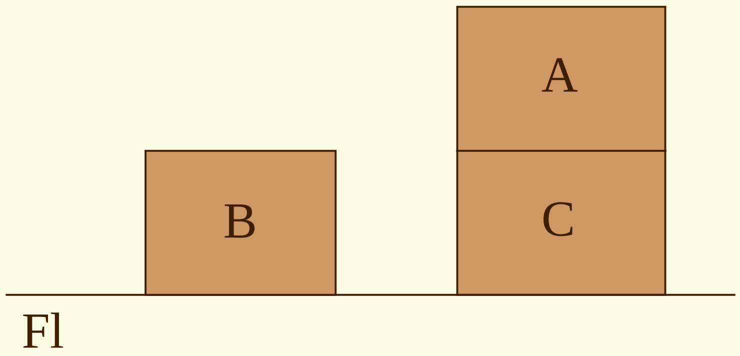
- Recipe for representing actions and their effects:
  - Reify the actions
  - Use function constant ‘do’ denoting a function mapping actions and states to states
  - Express the (positive and negative) effects of actions by wffs

Actions in situation calculus

- For example:
  - $[On(x,y,s) \wedge Clear(x,s) \wedge Clear(z,s) \wedge (x \neq z) \rightarrow On(x,z,do(move(x,y,z),s))]$
  - $[On(x,y,s) \wedge Clear(x,s) \wedge Clear(z,s) \wedge (x \neq z) \rightarrow \neg On(x,y,do(move(x,y,z),s))]$
  - $[On(x,y,s) \wedge Clear(x,s) \wedge Clear(z,s) \wedge (x \neq z) \wedge (y \neq z) \rightarrow Clear(y,do(move(x,y,z),s))]$
  - $[On(x,y,s) \wedge Clear(x,s) \wedge Clear(z,s) \wedge (x \neq z) \wedge (z \neq Fl) \rightarrow \neg Clear(z,do(move(x,y,z),s))]$

Actions in situation calculus

- Derived results: after applying move(B,A,Fl) it holds that:
  - On(B,Fl,do(move(B,A,Fl),S0))
  - ¬On(B,A,do(move(B,A,Fl),S0))
  - Clear(A,do(move(B,A,Fl),S0))



Frame axioms in situation calculus

- Frame axioms pertain to non-effects
  - $[On(x,y,s) \wedge (x \neq u)] \rightarrow On(x,y,do(move(u,v,z),s))$
  - $[\neg On(x,y,s) \wedge [(x \neq u) \vee (y \neq z)] \rightarrow \neg On(x,y,do(move(u,v,z),s))$
  - $Clear(u,s) \wedge (u \neq z) \rightarrow Clear(u,do(move(x,y,z),s))$
  - $\neg Clear(u,s) \wedge (u \neq y) \rightarrow \neg Clear(u,do(move(x,y,z),s))$

The Frame Problem

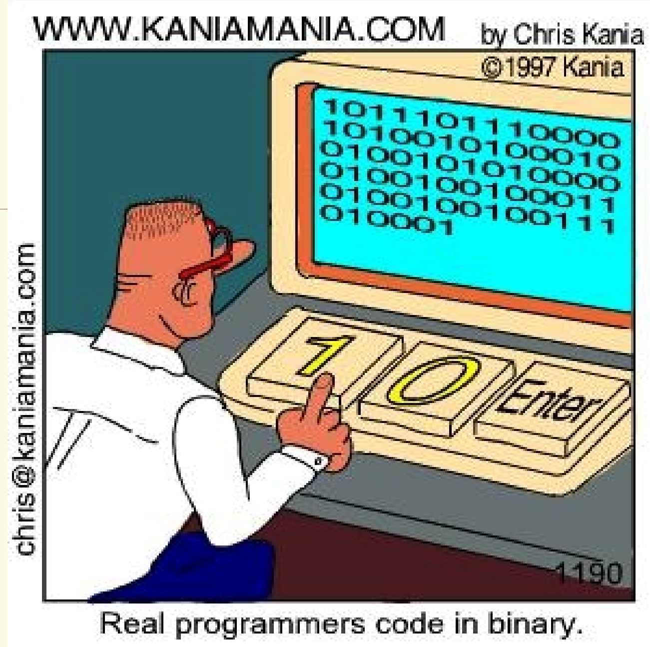
- Frame axioms are used to prove that a property of a state remains true if the state is changed by an action that *doesn't affect* that property
- In principle a pair of frame axioms is needed for every combination of fluent and action
- ***This is unmanageable in practice!!***

Relation with non-monotonic reasoning

- Historically, the study of the frame problem gave rise to the field of non-monotonic reasoning (NMR)
- E.g., persistence in terms of *default rule*

$$\frac{Holds(p(x),s) : \neg Ends(p(x), Result(A,s))}{Holds(p(x), Result(A,s))}$$

Agent-programming



AGENT0, PLACA, CONGOLOG, 3APL...

West; robot(x,y) <-
robot(x0,y0) AND rock(x0 - 1, y0) |
[IF y0 < y THEN noth ELSE south]; robot (x,y)

3APL

- A 3APL agent consists of :
  - a complex mental state incorporating
    - beliefs about the agent's environment
    - goals, representing the plans and states of affairs to be achieved
  - set of mechanisms working on mental state
    - to execute goals (controlling the environment)
    - for decision-making or practical reasoning (means-end reasoning and goal revision)
  - a set of capabilities, i.e. basic actions

3APL

- A 3APL agent program consists of :
  - a set of capabilities: basic actions:
    - e.g. `gripper_up, pickup, move_left, move_right, sense`
  - an initial belief base: simple propositions:
    - e.g. `block_on_table`
  - a set of initial goals: imperative-like programs
    - e.g. `gripper_up ; pickup`

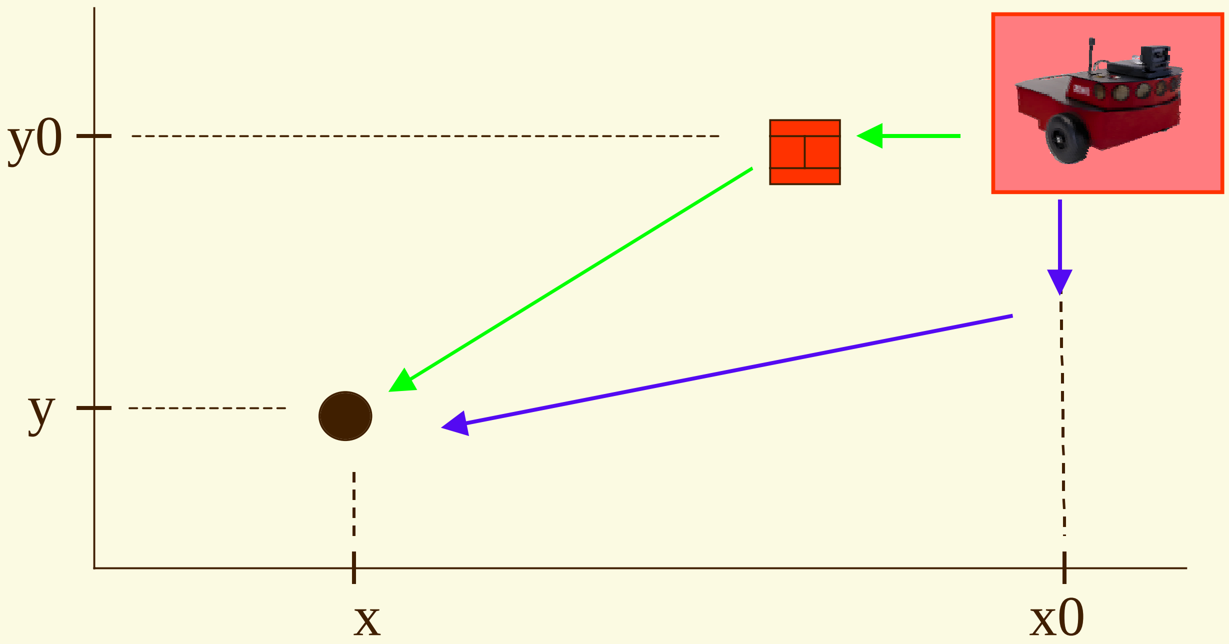
3APL

- a set of practical reasoning rules: guarded clauses of the form $\pi \leftarrow \varphi | \pi'$, where
  - π is a goal,
  - φ is a guard and
  - π' is a plan for goal π:
- e.g. `gripper_up;pickup ← no_block |        revise_goal`
- If the guard is implied by the agent's belief base the rule becomes applicable and *may* be applied.

3APL

- Example of *Practical reasoning rule*

```
west; robot(x,y) <-
robot(x0,y0) AND stone(x0-1, y0) |
[IF y0 < y THEN north ELSE south]; robot (x,y)
```



west; robot(x,y) ←

robot(x0,y0) AND steen(x0 - 1, y0) |

[IF y0 < y THEN noord ELSE zuid]; robot (x,y)